Departamento de Matemáticas

Facultad de Ciencias Naturales

Recinto de Río Piedras

MATE 3152

Apellidos: _____ _____ Nombre: _____

No. de estudiante: _____ Profesor: V. Keyantuo_____

Examen IV _____ 9 de mayo de 2012 _____ # de sección: 002 _____

Para obtener crédito muestre todo su trabajo. Explique claramente su contestación.

(1) (5 pts) Find the Taylor series of the function: $f(x) = \dfrac{5}{x+3}$ about the point $c = 5$.

(2) (10 pts.)

(a) (8 pts.)Find the Taylor series of the function: $f(x) = \dfrac{x^2+1}{x+3}$ about the point $c = 5$.
(*Hint. Start with a partial fraction decomposition*)

(b) (2 pts.)Find the radius of convergence of the power series obtained above.

(3) (5 pts.) Find an equation in polar coordinates for the line passing through $A(5,5)$ and having slope $m = -1$.

(4) (10 pts.)
    (a) (8 pts) Obtain the MacLaurin series for $f(x) = \sqrt{9 - 2x}$

(b) (2 pts.) Find the radius of convergence of the power series obtained above.

(5) (10 pts.) Obtain an equation in polar coordinates for the circle with equation $(x-3)^2+y^2 = 9$. (*Hint. Observe that it passes through the origin.*)

(6) (4 pts) A parabola has directrix $x = 10$ and focus at $(0,0)$. Obtain an equation in polar coordinates for the parabola.

(7) (4 pts) A parabola has directrix $y = 10$ and focus at $(0,0)$. Obtain an equation in polar coordinates for the parabola.

(8) (8 pts) An ellipse has eccentricity $e = \dfrac{1}{\pi}$ directrix $y = 16$ and a focus at the origin. Obtain an equation in polar coordinates.

(9) (6 pts) An ellipse has eccentricity $e = \dfrac{1}{\pi}$ directrix $x = -16$ and a focus at the origin. Obtain an equation in Cartesian coordinates.

(10) (30 pts) Let $f(x) = \dfrac{x^2}{\sqrt{4+x}}$

(a) (2 pts) Find the domain of definition of f.

(b) (2 pts) Compute $f'(x)$

(c) (2 pts) Compute $f''(x)$

(d) (2 pts) Compute $f'''(x)$

(e) (5 pts) Taylor series about $c = 2$ and the corresponding radius of convergence.

(f) (8 pts) Suppose $a \in D_f$, $a \neq -4$. Find the Taylor series of f about $x = a$.

(g) (2 pts.) Find the radius of convergence of the power series (about $x = a$) obtained above.

(h) (2 pts) Use the above results to obtain $f^{(6)}(12)$.

(i) (5 pts) Let $k \in \mathbb{N}$, and a be as above. Use the above results to obtain $f^{(k)}(a)$.

(11) (6 pts) Find the power series for $f(x) = \tan^{-1}(5x)$ about $x = 0$ (MacLaurin series).

(12) (6 pts) Find the power series for $f(x) = \sin(\pi x)$ about $x = 0$.

(13) (6 pts) Obtain the Taylor series for $f(x) = 5^x$ about $x = 2\ln(5)$. (Hint. Use the definition of the exponential function with base a, $a > 0$.)